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MCA-003-1162002

Seat No. ________

M. Sc. (Sem. II) (CBCS) Examination

April / May - 2018

Mathematics : 2002

(Complex Analysis)

Faculty Code : 003

Subject Code : 1162002

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are five questions.

(2) All questions are compulsory.

(2) Each questions carries 14 marks.

1 Do as directed : (Attempt Any Seven) **14**

(a) Evaluate $\int_{\sigma} \frac{e^{iz}}{z^2} dz$ where $\sigma(t) = e^{it}; t \in [0, 2\pi]$.

(b) Define : (i) Pole. (ii) Essential singularity.

(c) State necessary and sufficient condition for an isolated singularity to be a removable singularity. Also mention the type of singularity for $f(z) = e^{\frac{1}{z}}$.

(d) State the geometric meaning of winding number for the closed rectifiable curve.

(e) (i) Define conformal mapping.

(ii) State Inverse function theorem.

(f) State Riemann stieltje's theorem.

(g) Evaluate $\int_{\sigma} \frac{z^2+1}{z^2+z+1} dz$; where $|z| = 2$ is the circle with center 0 and radius 2.

(h) Find bilinear transformation taking $i \to 1, 0 \to \infty, -i \to 0$.

2 Attempt any **two** of the following : **14**

(a) Define branch of logarithmic on a connected open set and prove that if $f : G \to \mathbb{C}$ be continuous, $g : H \to \mathbb{C}$ be differentiable with $g'(x) \neq 0; \forall x \in H$ and $f(G) \subset H, g(f(z)) = z; z \in G$ then f is differentiable and $f'(z) = \dfrac{1}{g^r(f(z))}; z \in G.$ **7**

(b) (i) Prove that $e^{z+w} = e^z . e^w; z, w \in \mathbb{C}.$ **4**

(ii) Justify with an example that $Log(z_1 z_2) \neq Log(z_1) + Log(z_2).$ **3**

(c) (i) Give an example which shows that Cauchy Riemann equations are merely necessary nut not sufficient. **2**

(ii) Prove that for an analytic function $f : G \to \mathbb{C};$ where G be an open connected subset of $\mathbb{C}$ and $G^* = \{\overline{z} / z \in G\}$ then $f^* : G^* \to \mathbb{C}$ defined by $f^*(z) = \overline{f(\overline{z})}; z \in G^*$ is analytic. **5**

3 All are compulsory : **14**

(a) Show that the set M = { S / S is a bilinear transformation} is a group under composition. **7**

(b) (i) State and prove Liouvilles Theorem. **3**

(ii) Prove that if $\gamma : [a, b] \to \mathbb{C}$ be a function of bounded variation with $a < c < b$ then $\gamma|[a, c] : [a, c] \to \mathbb{C}$ and $\gamma|[c, b] : [c, b] \to \mathbb{C}$ are function of bounded variation and $V(\gamma) = V(\gamma|[a, c]) + V(\gamma|[c, b])$ **4**

OR

3 All are compulsory : **14**

(a) Every bilinear transformation can be written as composition of translation, dilation and inversion. **7**

(b) (i) State and prove Open Mapping Theorem. **4**

(ii) Evaluate $\int_{\sigma} \frac{dz}{z^2 + i^2}$, where σ is given by $\sigma(t) = 2e^{it} \left| \cos 2t \right|$. **3**

4 Attempt any **two** of the following : **14**

(a) State and prove Fundamental theorem of algebra. **7**

(b) State and prove Minimum modulus theorem. Also give an example of a non-constant analytic function in $\mathbb{C}$ which may attains its minimum value but not maximum. **7**

(c) Prove that if $f : G - \{a\} \to \mathbb{C}$ be an analytic function and α is a pole of f then there exists $m \in \mathbb{N}$ and $g : G \to \mathbb{C}$ such that $f(z) = \frac{g(z)}{(z-a)^m}; \forall \alpha = z.$ **7**

(d) State and prove Cauchy's Theorem. **7**

5 Attempt any **two** of the following : **14**

(a) State and prove Cauchy's Integral formula for second version. **7**

(b) Prove that every z^m of f has a finite order multiplicity. **7**

(c) Find Laurent's series expansion in the powers of z for $f(z) = \frac{z+2}{z^2 - 2z - 3}$ in **7**

(i) $|z| < 1;$

(ii) $1 < |z| < 3;$

(iii) $|z| > 3.$

(d) State orientation Principle. Also show the concept of symmetric point with respect to a circle Γ is independent of choice of three points $z_2, z_3, z_4 \in \Gamma$. **7**